Theta 2D Geometry

For each question, "E) NOTA" indicates that none of the above answers is correct.

- 1. Find the area of the triangular region formed by connecting the points (0, 10), (5, 5), and the origin.
- A) 50 B) 25 C) 15 D) 10 E) NOTA

2. In  $\triangle ABC$ , *D* and *E* are midpoints of  $\overline{AB}$  and  $\overline{BC}$ , respectively. If  $\overline{CD}$  intersects  $\overline{AE}$  at point *P*, find  $\frac{CP}{CD}$ .

- A) 1:1 B) 2:1 C) 2:3 D) 1:2 E) NOTA
- 3. How many scalene triangles having all sides of integral length can be formed if the perimeter must be less than 12?

4. Find the area of kite *ABCD* if  $m \angle ABC = 120^{\circ}$ ,  $\angle BAD$  and  $\angle BCD$  are right angles, and BC = 3. A)  $5\sqrt{3}$  B)  $3\sqrt{3}$  C)  $\frac{9\sqrt{3}}{2}$  D)  $9\sqrt{3}$  E) NOTA

5. The sum of the measures of the interior angles of a convex polygon is 4860°. How many sides does the polygon have?

A) 25 B) 27 C) 28 D) 31 E) NOTA

6. Circle *M* and circle *J* are externally tangent. If the radius of circle *M* is 8 and the radius of circle *J* is 12, find the length of the common external tangent of circle *M* and circle *J* (the distance between the points of tangency).

- A)  $20\sqrt{2}$  B)  $8\sqrt{6}$  C)  $8\sqrt{3}$  D)  $6\sqrt{10}$  E) NOTA
- 7. If  $a \parallel b, m \angle 1 = (10x + 27)^\circ$ , and  $m \angle 6 = (12x + 21)^\circ$ , find  $m \angle 8$ .



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8. Points A, B, d	C, and D are collinea	ar. Point <i>B</i> is the midp	point of $\overline{AD}$ and po	int C is the midpoint	
of BD. AB	is 12 less than 8 tim	es BC. Find AD.			
A) 2	B) 4	C) 8	D) 16	E) NOTA	
9. If p is false, o	q is true, and r is tru	e, then which of the f	ollowing statement	s is/are true?	
I. $(p \lor q) \land r$		II. <i>p</i> ∨( <i>q</i> ∧ <i>r</i> )	II. $p \lor (q \land r)$ III. $(p \land r) \lor (p \land q)$		
A) I only	B) II only	C) I and II only	D) I, II, and III	E) NOTA	
10. If two lines	do not intersect, th	en			
A) the lines are	always parallel.	E	<ol><li>the lines are alw</li></ol>	ays skew.	
C) the lines are	sometimes coplana	ar. C	D) the lines are never parallel.		

- E) NOTA
- 11. The lengths of the sides of a triangle are 5, 6, and 9. Find the length of the altitude constructed to the shortest side.

A) 
$$\frac{20\sqrt{2}}{9}$$
 B)  $\frac{10\sqrt{2}}{3}$  C)  $4\sqrt{2}$  D)  $10\sqrt{2}$  E) NOTA

12. The circle is inscribed in  $\triangle ABC$  and is tangent to  $\overline{BC}$  at point *M*. If AB = 5, AC = 6, and BC = 7, then find *BM*.



- 13. A circle has a radius of 3. Find the perimeter of the square inscribed in the circle.
- A)  $3\sqrt{2}$  B) 6 C)  $12\sqrt{2}$  D) 18 E) NOTA
- 14. Negate the statement, "All geometry students are freshmen."
- A) Some geometry students are not freshmen.
- B) Some geometry students are freshmen.
- C) No geometry students are freshmen.
- D) At least one geometry student is a freshman.
- E) NOTA

- 15. The base of a triangle is 4 times as long as the side of a square. If the triangle and the square have equal enclosed areas, find the ratio of an altitude (to the described base) of the triangle to the side of the square.
- A) 1:4 B) 1:2 C) 2:1 D) 4:1 E) NOTA

16. In  $\triangle ABC$ , BC = 6, AC = 9, and  $m \angle ACB = 60^{\circ}$ . Find the area enclosed by the triangle.

A) 
$$\frac{27\sqrt{3}}{2}$$
 B)  $\frac{27}{2}$  C)  $\frac{18\sqrt{95}}{5}$  D)  $\frac{25\sqrt{7+2\sqrt{3}}}{2}$  E) NOTA

17. The dartboard pictured consists of 3 concentric circles with radii 2, 4, and 6. Find the probability that a dart thrown that hits the dartboard hits the region worth 10 points.



18. In  $\triangle ABC$ , point *E* is on  $\overline{AB}$  and point *D* is on  $\overline{BC}$ . Let  $\overline{AD} \cap \overline{CE} = \{F\}$ . If  $\frac{AE}{EB} = \frac{1}{3}$  and  $\frac{CD}{DB} = \frac{1}{2}$ , then find  $\frac{EF}{FC} + \frac{AF}{FD}$ . A)  $\frac{1}{2}$ B)  $\frac{4}{5}$ C)  $\frac{3}{2}$ D) 2
E) NOTA 19. In circle *C*, the area of a sector is  $20\pi$  and the central angle of the sector measures  $72^{\circ}$ . Find the length of the radius of circle *C*.

A) 10 B) 20 C) 50 D) 100 E) NOTA

20.	Find the y-inte	rcept of the Euler li	ne of the triangle w	ith vertices (1,-1),	(1,5), and (4,5)
A)	7	B) 3	C) 2	D) —2	E) NOTA

21. BC,CE, and AE are tangents to circle G at B, D, and F, respectively. If

 $mBH = 96^{\circ}$ , BC = 5, CE = 7, and BG = 3, find the perimeter of quadrilateral BFEC.



22. Find the area of the shaded lune in circle *C*, where length  $6\sqrt{2}$  is a radius of the circle not shown in the picture.



- 23. Suppose that a line intersects two sides of an equilateral triangle such that the line is parallel to the third side of the triangle. If the line divides the equilateral triangle into a trapezoid and a smaller triangle and if the trapezoid and smaller triangle have the same perimeter, find the ratio of the area enclosed by the trapezoid to the area enclosed by the smaller triangle.
- A) 2:3 B) 3:4 C) 9:16 D) 7:9 E) NOTA
- 24. In  $\triangle ABC$ ,  $m \angle ABC = 150^\circ$ ,  $AB = 3\sqrt{3}$ , and BC = 4. If a perpendicular line is constructed to  $\overline{AB}$  through point A and a perpendicular line is constructed to  $\overline{BC}$  through point C, the perpendicular lines intersect at point D. Find the area enclosed by the quadrilateral ABCD.
- A)  $13\sqrt{3} + 21$  B)  $50\sqrt{3}$  C)  $\frac{91\sqrt{3}}{2}$  D)  $\frac{109\sqrt{3}}{2}$  E) NOTA
- 25. Two tangents are drawn to a circle from an exterior point *C*. The tangents touch the circle at points *A* and *B*. A third tangent intersects the circle at point *X*, intersects  $\overline{BC}$  at point *Y*, and intersects  $\overline{AC}$  at point *Z*. If AC = 20, then find the perimeter of  $\Delta CYZ$ .
- A) 20 B) 30 C) 40 D) impossible to E) NOTA determine

- 26. A triangle is inscribed in a semicircle. The length of the longest side of the triangle is twice as long as the length of the shortest side of the triangle. The semicircle has a circumference of  $12\pi + 24$ . If a circle is inscribed in the right triangle, find the radius of the inscribed circle.
- A)  $6\sqrt{3}-6$  B) 12 C)  $12\sqrt{3}$  D)  $12\sqrt{3}-12$  E) NOTA
- 27. Find the image of the point (-5, 7) after a 90° counterclockwise rotation about the origin.
- A) (5, 7) B) (-5, -7) C) (7, 5) D) (-7, -5) E) NOTA
- 28. In rectangle ABCD,  $\overline{AC}$ ,  $\overline{BD}$ ,  $\overline{GH}$  intersect at point *E*. If AB = 12 and AD = 4, find the area of the shaded region.



- 29. A trapezoid has bases that measure 5 and 13. The legs of the trapezoid measure  $2\sqrt{21}$  and 10. Find the height of the trapezoid.
- A)  $2\sqrt{3}$  B)  $2\sqrt{6}$  C)  $5\sqrt{3}$  D)  $5\sqrt{6}$  E) NOTA
- 30. In circle *E*,  $\overline{AB} \cong \overline{CD}$ . Find *AB*.



$\alpha \rightarrow \sqrt{41}$	18 (ח	F) ΝΟΤΔ
C) ZV4I	0,10	